# To the Nonlocal Theory of Waves in Physical Vacuum 

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#### Abstract

The wave transport processes in physical vacuum are investigated. The corresponding equations of nonlocal physical description are obtained. The vast mathematical modeling is realized for different values of parameter of nonlocality. The self-consistent force acting on the volume unit is calculated.


Keywords: Nonlocal theory of transport processes, physical vacuum, wave processes in physical vacuum.

## 1 Introduction

In monographs [1-4] the evolution of Physical Vacuum (PV) in Planck epoch and wave effects in PV are considered in the frame of nonlocal physics. It should be underlined that the nonlocal transport equations are obtained from the first principles of physics. Here we intend to find the wave solutions of PV equations as a particular case of the nonlocal theory of gravitational waves.

During all investigations we needn't to use the theory Newtonian gravitation for solution of nonlinear non-local evolution equations. In contrast with the local physics this approach in the frame of quantum non-local hydrodynamics leads to the closed mathematical description for the physical system under consideration. If the matter is absent, non-local evolution equations have nevertheless non-trivial solutions corresponding evolution of PV which description in time and 3D space on the level of quantum hydrodynamics demands only quantum pressure $p$, the self-consistent force $\mathbf{F}$ (acting on unit of the space volume) and velocity $\mathbf{v}_{\mathbf{0}}$.

We intend to find the solutions of the transport equations defining the evolution the physical vacuum (PV). It means:

1. The system of non local equations should be written for the case when the usual matter is absent ( $\rho=0$ ), also radiation, gravitation (as well as other mass forces) and electromagnetic fields are absent.
2. No reason to speak about special or general relativity in this situation, because these theories don't work in the described conditions.
3. Formally speaking the Newtonian gravity propagates with the infinite speed. This conclusion is connected only with the description in the frame of local physics. Usual affirmation - general relativity (GR) reduces to Newtonian gravity in the weak-field, low-velocity limit. In literature you can find criticism of this affirmation because the conservation of angular momentum is implicit in the assumptions on which GR rests. Finite propagation speeds and conservation of angular momentum are incompatible in GR. Therefore, GR was forced to claim that gravity is not a force that propagates in any classical sense, and that aberration does not apply. But here I do not intend to join in this widely discussed topic using only unified non-local model.
4. Physical Vacuum is "one species" system.

Let us apply generalized quantum hydrodynamic equations for investigation of the evolution PV using (for better understanding) non-stationary 1D Cartesian description.

First of all we call attention to the fact that nonlocal equations contain two forces of gravitational origin, $\mathbf{F}$ - the force acting on the unit volume of the space and $\mathbf{g}$ - the force acting on the unit mass. As a result we have nonlocal equations [1-4]:
(continuity equation)

$$
\begin{equation*}
\frac{\partial}{\partial t}\left\{\rho-\tau\left[\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial \mathbf{r}} \cdot\left(\rho \mathbf{v}_{0}\right)\right]\right\}+\frac{\partial}{\partial \mathbf{r}} \cdot\left\{\rho \mathbf{v}_{0}-\tau\left[\frac{\partial}{\partial t}\left(\rho \mathbf{v}_{0}\right)+\frac{\partial}{\partial \mathbf{r}} \cdot\left(\rho \mathbf{v}_{0} \mathbf{v}_{0}\right)+\overrightarrow{\mathrm{I}} \cdot \frac{\partial p}{\partial \mathbf{r}}-\mathbf{F}\right]\right\}=0 \tag{1.1}
\end{equation*}
$$

(continuity equation, 1D case)

$$
\begin{equation*}
\frac{\partial}{\partial t}\left\{\rho-\tau\left[\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)\right]\right\}+\frac{\partial}{\partial x}\left\{\rho u-\tau\left[\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}\left(\rho u^{2}\right)+\frac{\partial p}{\partial x}-F\right]\right\}=0 \tag{1.2}
\end{equation*}
$$

(momentum equation)

$$
\begin{align*}
& \frac{\partial}{\partial t}\left\{\rho \mathbf{v}_{0}-\tau\left[\frac{\partial}{\partial t}\left(\rho \mathbf{v}_{0}\right)+\frac{\partial}{\partial \mathbf{r}} \cdot \rho \mathbf{v}_{0} \mathbf{v}_{0}+\frac{\partial p}{\partial \mathbf{r}}-\mathbf{F}\right]\right\}-\mathbf{g}\left[\rho-\tau\left(\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial \mathbf{r}} \cdot\left(\rho \mathbf{v}_{0}\right)\right)\right] \\
& +\frac{\partial}{\partial \mathbf{r}} \cdot\left\{\rho \mathbf{v}_{0} \mathbf{v}_{0}+p \overrightarrow{\mathrm{I}}-\tau\left[\begin{array}{l}
\left.\frac{\partial}{\partial t}\left(\rho \mathbf{v}_{0} \mathbf{v}_{0}+p \overrightarrow{\mathrm{I}}\right)+\frac{\partial}{\partial \mathbf{r}} \cdot \rho\left(\mathbf{v}_{0} \mathbf{v}_{0}\right) \mathbf{v}_{0}+2 \overrightarrow{\mathrm{I}}\left(\frac{\partial}{\partial \mathbf{r}} \cdot\left(p \mathbf{v}_{0}\right)\right)\right] \\
+\frac{\partial}{\partial \mathbf{r}} \cdot\left(\overrightarrow{\mathrm{I}} p \mathbf{v}_{0}\right)-\mathbf{F} \mathbf{v}_{0}-\mathbf{v}_{0} \mathbf{F}
\end{array}\right]=0\right. \tag{1.3}
\end{align*}
$$

(momentum equation, 1D case)

$$
\begin{align*}
& \frac{\partial}{\partial t}\left\{\rho u-\tau\left[\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}\left(\rho u^{2}\right)+\frac{\partial p}{\partial x}-F\right]\right\} \\
& -g\left[\rho-\tau\left(\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)\right)\right]  \tag{1.4}\\
& +\frac{\partial}{\partial x}\left\{\rho u^{2}+p-\tau\left[\frac{\partial}{\partial t}\left(\rho u^{2}+p\right)+\frac{\partial}{\partial x}\left(\rho u^{3}+3 p u\right)-2 F u\right]\right\}=0
\end{align*}
$$

(energy equation)

$$
\begin{align*}
& \frac{\partial}{\partial t}\left\{\frac{\rho v_{0}^{2}}{2}+\frac{3}{2} p-\tau\left[\frac{\partial}{\partial t}\left(\frac{\rho v_{0}^{2}}{2}+\frac{3}{2} p\right)\right.\right. \\
& \left.\left.+\frac{\partial}{\partial \mathbf{r}} \cdot\left(\frac{1}{2} \rho v_{0}^{2} \mathbf{v}_{0}+\frac{5}{2} p \mathbf{v}_{0}\right)-\mathbf{F} \cdot \mathbf{v}_{0}\right]\right\} \\
& +\frac{\partial}{\partial \mathbf{r}} \cdot\left\{\frac{1}{2} \rho v_{0}^{2} \mathbf{v}_{0}+\frac{5}{2} p \mathbf{v}_{0}-\tau\left[\frac { \partial } { \partial t } \left(\frac{1}{2} \rho v_{0}^{2} \mathbf{v}_{0}\right.\right.\right. \\
& \left.+\frac{5}{2} p \mathbf{v}_{0}\right)+\frac{\partial}{\partial \mathbf{r}} \cdot\left(\frac{1}{2} \rho v_{0}^{2} \mathbf{v}_{0} \mathbf{v}_{0}+\frac{7}{2} p \mathbf{v}_{0} \mathbf{v}_{0}+\frac{1}{2} p v_{0}^{2} \overline{\mathrm{I}}\right.  \tag{1.5}\\
& \left.\left.\left.+\frac{5}{2} \frac{p^{2}}{\rho} \tilde{\mathrm{I}}\right)-\mathbf{F} \cdot \mathbf{v}_{0} \mathbf{v}_{0}-p \mathbf{g} \cdot \overrightarrow{\mathrm{I}}-\frac{1}{2} v_{0}^{2} \mathbf{F}-\frac{3}{2} \mathbf{g} p\right]\right\} \\
& -\left\{\mathbf{F} \cdot \mathbf{v}_{0}-\tau\left[\mathbf { g } \cdot \left(\frac{\partial}{\partial t}\left(\rho \mathbf{v}_{0}\right)+\frac{\partial}{\partial \mathbf{r}} \cdot \rho \mathbf{v}_{0} \mathbf{v}_{0}\right.\right.\right. \\
& \left.\left.\left.+\frac{\partial}{\partial \mathbf{r}} \cdot p \ddot{\mathbf{I}}-\mathbf{F}\right)\right]\right\}=0
\end{align*}
$$

(energy equation, 1D case)

$$
\begin{align*}
& \frac{\partial}{\partial t}\left\{\rho u^{2}+3 p-\tau\left[\frac{\partial}{\partial t}\left(\rho u^{2}+3 p\right)+\frac{\partial}{\partial x}\left(\rho u^{3}+5 p u\right)-2 F u\right]\right\} \\
& +\frac{\partial}{\partial x}\left\{\rho u^{3}+5 p u-\tau\left[\frac{\partial}{\partial t}\left(\rho u^{3}+5 p u\right)+\frac{\partial}{\partial x}\left(\rho u^{4}+8 p u^{2}\right)\right.\right. \\
& \left.\left.-2 F u^{2}-u^{2} F\right]\right\}+5 \frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho} F-\frac{\partial}{\partial x} \frac{p^{2}}{\rho}\right)\right\}  \tag{1.6}\\
& -2\left\{F u-\tau g\left(\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}\left(\rho u^{2}\right)+\frac{\partial p}{\partial x}-F\right)\right\}=0
\end{align*}
$$

Here $p$ is pressure of PV, $u$ is velocity of PV expanding, and $F$ is the self consistent force acting in PV, $\tau$ is nonlocal parameter. Nonlinear evolution equations (1.1) - (1.6) contain the forces $\mathbf{F}, \mathbf{g}$ acting on space and masses including cross-term (see for example the last line in equation (1.6)). The relation $\mathbf{F}=\rho \mathbf{g}$ comes into being only after the mass appearance as a result of the PV explosion.
Now we consider the limit case $\rho \rightarrow 0$ corresponding to transfer to Physical Vacuum in 1D case. We have:

Continuity equation $\rho \rightarrow 0$ (see (1.2))

$$
\begin{equation*}
\frac{\partial}{\partial x}\left\{\tau\left[\frac{\partial p}{\partial x}-F\right]\right\}=0 \tag{1.7}
\end{equation*}
$$

Momentum equation $\rho \rightarrow 0$ (see (1.4))

$$
\begin{equation*}
\frac{\partial}{\partial t}\left\{\tau\left[\frac{\partial p}{\partial x}-F\right]\right\}-\left(\frac{\partial p}{\partial x}-F\right)+\frac{\partial}{\partial x}\left\{\tau\left[\frac{\partial p}{\partial t}+3 \frac{\partial}{\partial x}\left(p v_{0}\right)-2 F v_{0}\right]\right\}=0 \tag{1.8}
\end{equation*}
$$

Energy equation $\rho \rightarrow 0$ (see (1.6)). We deliver some intermediate transformations.

$$
\begin{align*}
& 3 \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right) \\
& -\frac{\partial}{\partial t}\left\{\tau\left[3 \frac{\partial p}{\partial t}+5 p \frac{\partial u}{\partial x}+3 u \frac{\partial p}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right)\right]\right\} \\
& -\frac{\partial}{\partial x}\left\{\tau\left[5 \frac{\partial}{\partial t}(p u)+8 \frac{\partial}{\partial x}\left(p u^{2}\right)-3 F u^{2}\right]\right\}  \tag{1.9}\\
& +5 \frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho} F-\frac{\partial}{\partial x} \frac{p^{2}}{\rho}\right)\right\}=0
\end{align*}
$$

or

$$
\begin{align*}
& 3 \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right) \\
& -\frac{\partial}{\partial t}\left\{\tau\left[3 \frac{\partial p}{\partial t}+5 p \frac{\partial u}{\partial x}+3 u \frac{\partial p}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right)\right]\right\} \\
& -\frac{\partial}{\partial x}\left\{\tau\left[5 \frac{\partial}{\partial t}(p u)+8 u^{2} \frac{\partial p}{\partial x}+8 p \frac{\partial u^{2}}{\partial x}-3 F u^{2}\right]\right\}  \tag{1.10}\\
& +5 \frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho} F-\frac{\partial}{\partial x} \frac{p^{2}}{\rho}\right)\right\}=0 \\
3 & \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right) \\
- & \frac{\partial}{\partial t}\left\{\tau\left[3 \frac{\partial p}{\partial t}+5 p \frac{\partial u}{\partial x}+3 u \frac{\partial p}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right)\right]\right\} \\
- & \frac{\partial}{\partial x}\left\{\tau\left[5 \frac{\partial}{\partial t}(p u)+5 u^{2} \frac{\partial p}{\partial x}+3 u^{2}\left(\frac{\partial p}{\partial x}-F\right)+8 p \frac{\partial u^{2}}{\partial x}\right]\right\}  \tag{1.11}\\
+ & \frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho} F-\frac{\partial}{\partial x} \frac{p^{2}}{\rho}\right)\right\}=0
\end{align*}
$$

or

$$
\begin{align*}
& 3 \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right) \\
& -\frac{\partial}{\partial t}\left\{\tau\left[3 \frac{\partial p}{\partial t}+5 p \frac{\partial u}{\partial x}+3 u \frac{\partial p}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right)\right]\right\} \\
& -\frac{\partial}{\partial x}\left\{\tau\left[5 \frac{\partial}{\partial t}(p u)+5 u^{2} \frac{\partial p}{\partial x}+16 p u \frac{\partial u}{\partial x}\right]\right\}  \tag{1.12}\\
& -\frac{\partial}{\partial x}\left\{\tau\left[3 u^{2}\left(\frac{\partial p}{\partial x}-F\right)\right]\right. \\
& +5 \frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho} F-\frac{\partial}{\partial x} \frac{p^{2}}{\rho}\right)\right\}=0
\end{align*}
$$

and finally

$$
\begin{align*}
& 3 \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right) \\
& -\frac{\partial}{\partial t}\left\{\tau\left[3 \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right)\right]\right\}  \tag{1.13}\\
& -\frac{\partial}{\partial x}\left\{\tau\left[5 \frac{\partial}{\partial t}(p u)+5 u \frac{\partial}{\partial x}(p u)+11 u p \frac{\partial u}{\partial x}\right]\right\} \\
& -6 \tau\left(\frac{\partial p}{\partial x}-F\right) u \frac{\partial u}{\partial x}+A^{\text {pert }}=0
\end{align*}
$$

In the energy equation (1.13) the term $A^{\text {pert }}$ defines the possible perturbation of physical vacuum as a result of the perturbation appearance like Higgs boson; $A^{\text {pert }}$ can be estimated by different ways (see also [4]).

$$
\begin{equation*}
A^{\text {pert }}=5 \frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho} F-\frac{\partial}{\partial x} \frac{p^{2}}{\rho}\right)\right\} \tag{1.14}
\end{equation*}
$$

Then the system of equations defining the PV evolution for this case can be written as follows ( $u$ velocity in the $x$-direction):
"System 1"
(continuity equation, 1D case)

$$
\begin{equation*}
\frac{\partial}{\partial x}\left\{\tau\left[\frac{\partial p}{\partial x}-F\right]\right\}=0 \tag{1.15}
\end{equation*}
$$

(momentum equation, 1D case)

$$
\begin{align*}
& \frac{\partial p}{\partial x}-F-\frac{\partial}{\partial t}\left\{\tau\left[\frac{\partial p}{\partial x}-F\right]\right\}-2 \tau\left(\frac{\partial p}{\partial x}-F\right) \frac{\partial u}{\partial x} \\
& -\frac{\partial}{\partial x}\left\{\tau\left[\frac{\partial p}{\partial t}+u \frac{\partial p}{\partial x}+3 p \frac{\partial u}{\partial x}\right]\right\}=0 \tag{1.16}
\end{align*}
$$

(energy equation, 1D case)

$$
\begin{align*}
& 3 \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right) \\
& -\frac{\partial}{\partial t}\left\{\tau\left[3 \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right)\right]\right\}  \tag{1.17}\\
& -\frac{\partial}{\partial x}\left\{\tau\left[5 \frac{\partial}{\partial t}(p u)+5 u \frac{\partial}{\partial x}(p u)+11 u p \frac{\partial u}{\partial x}\right]\right\} \\
& -6 \tau\left(\frac{\partial p}{\partial x}-F\right) u \frac{\partial u}{\partial x}+A^{\text {pert }}=0
\end{align*}
$$

The value $A^{\text {pert }}$ can be considered as an external fluctuation. Let us transform (1.17)

$$
\begin{align*}
& A^{\text {pert }}=5 \frac{\partial}{\partial x}\left\{\tau\left(\frac{p}{\rho} F-\frac{\partial}{\partial x} \frac{p^{2}}{\rho}\right)\right\}=5 \frac{\partial}{\partial x}\left[\tau\left(\frac{p}{\rho} F-\frac{p}{\rho} \frac{\partial p}{\partial x}\right)\right]-5 \frac{\partial}{\partial x}\left[\tau\left(p \frac{\partial}{\partial x} \frac{p}{\rho}\right)\right] \\
& =5 \frac{\partial}{\partial x}\left[\tau \frac{p}{\rho}\left(F-\frac{\partial p}{\partial x}\right)\right]-5 \frac{\partial}{\partial x}\left[\tau p \frac{\partial}{\partial x} \frac{p}{\rho}\right]=  \tag{1.18}\\
& =5 \frac{p}{\rho} \frac{\partial}{\partial x}\left[\tau\left(F-\frac{\partial p}{\partial x}\right)\right]+5 \tau\left(F-\frac{\partial p}{\partial x}\right) \frac{\partial}{\partial x}\left[\frac{p}{\rho}\right]-5 \frac{\partial}{\partial x}\left[\tau p \frac{\partial}{\partial x} \frac{p}{\rho}\right]
\end{align*}
$$

Relation (1.18) can be simplified using (1.15)

$$
\begin{equation*}
A^{\text {pert }}=5 \tau\left(F-\frac{\partial p}{\partial x}\right) \frac{\partial}{\partial x}\left[\frac{p}{\rho}\right]-5 \frac{\partial}{\partial x}\left[\tau p \frac{\partial}{\partial x} \frac{p}{\rho}\right] \tag{1.19}
\end{equation*}
$$

If the gradient of the initial energy perturbation is small

$$
\begin{equation*}
A^{\text {pert }} \cong 0 \tag{1.20}
\end{equation*}
$$

## 2 Wave Solutions of "System 1"

We intend to find the wave solutions of "System 1". Some significant remarks:

1. In principle we can use the spherical coordinate system considering the explosion of "PV bubbles" [4], but for our aims the non-stationary Cartesian 1D description is sufficient. For example it corresponds to flat gravitational wave from a remote source of the perturbations.
2. For simplicity and more transparent description we suppose that nonlocal parameter $\tau$ is constant. Moreover we intend to investigate the situation when the relation (1.20) is fulfilled.

As a result we have "System 2":
(continuity equation, 1D case)

$$
\begin{equation*}
\frac{\partial}{\partial x}\left\{\tau\left[\frac{\partial p}{\partial x}-F\right]\right\}=0 \tag{2.1}
\end{equation*}
$$

(momentum equation, 1D case)

$$
\begin{align*}
& \frac{\partial p}{\partial x}-F-\frac{\partial}{\partial t}\left\{\tau\left[\frac{\partial p}{\partial x}-F\right]\right\}-2 \tau\left(\frac{\partial p}{\partial x}-F\right) \frac{\partial u}{\partial x} \\
& -\frac{\partial}{\partial x}\left\{\tau\left[\frac{\partial p}{\partial t}+u \frac{\partial p}{\partial x}+3 p \frac{\partial u}{\partial x}\right]\right\}=0 \tag{2.2}
\end{align*}
$$

(energy equation, 1D case)

$$
\begin{align*}
& 3 \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right) \\
& -\frac{\partial}{\partial t}\left\{\tau\left[3 \frac{\partial p}{\partial t}+3 u \frac{\partial p}{\partial x}+5 p \frac{\partial u}{\partial x}+2 u\left(\frac{\partial p}{\partial x}-F\right)\right]\right\}  \tag{2.3}\\
& -\frac{\partial}{\partial x}\left\{\tau\left[5 \frac{\partial}{\partial t}(p u)+5 u \frac{\partial}{\partial x}(p u)+11 u p \frac{\partial u}{\partial x}\right]\right\} \\
& -6 \tau\left(\frac{\partial p}{\partial x}-F\right) u \frac{\partial u}{\partial x}=0
\end{align*}
$$

We investigate the possibility of the object formation of the traveling wave type. For this solution there is only dependence on $\xi$ as combination of coordinate $x$ and time $t$ without the explicit dependence on time. The validity of this assumption will be investigated.

We consider the case when $\xi=x+u t$.
The equations (2.1) - (2.3) take the form:
continuity equation

$$
\begin{equation*}
\frac{\partial}{\partial \xi}\left[\frac{\partial p}{\partial \xi}-F\right]=0 \tag{2.4}
\end{equation*}
$$

moment equation

$$
\begin{equation*}
\frac{\partial p}{\partial \xi}-F-2 \tau\left(\frac{\partial p}{\partial \xi}-F\right) \frac{\partial u}{\partial \xi}-3 \tau \frac{\partial}{\partial \xi}\left\{p \frac{\partial u}{\partial \xi}\right\}+2 \tau \frac{\partial}{\partial \xi}\left\{u \frac{\partial p}{\partial \xi}\right\}=0 \tag{2.5}
\end{equation*}
$$

Moment equation (2.5) is obtained with the help of Eq. (2.4), and integration leads to result:

$$
\frac{\partial p}{\partial \xi}-F=C
$$

where $C$ is const. Then

$$
\begin{equation*}
C-2 \tau C \frac{\partial u}{\partial \xi}-3 \tau \frac{\partial}{\partial \xi}\left\{p \frac{\partial u}{\partial \xi}\right\}+2 \tau \frac{\partial}{\partial \xi}\left\{u \frac{\partial p}{\partial \xi}\right\}=0 \tag{2.6}
\end{equation*}
$$

Energy equation is written as

$$
\begin{align*}
& 6 u \frac{\partial p}{\partial \xi}+5 p \frac{\partial u}{\partial \xi}+2 u C-16 \tau u \frac{\partial}{\partial \xi}\left[u \frac{\partial p}{\partial \xi}\right]-26 \tau u \frac{\partial}{\partial \xi}\left[p \frac{\partial u}{\partial \xi}\right] \\
& -\tau\left[10 u \frac{\partial p}{\partial \xi}+21 p \frac{\partial u}{\partial \xi}+8 C u\right] \frac{\partial u}{\partial \xi}=0 \tag{2.7}
\end{align*}
$$

In the limit of local physics the system of equations $\tau=0$ is not a closed system.
Let us write down these equations in the dimensionless form, where dimensionless symbols are marked by tildes, using the introduced scales

$$
\begin{equation*}
u_{0}, \tau, p_{0},[\xi]=u_{0} \tau=\xi_{0},[F]=\frac{p_{0}}{u_{0} \tau}=C, \quad[C]=\frac{p_{0}}{u_{0} \tau}=\frac{p_{0}}{\xi_{0}} \tag{2.8}
\end{equation*}
$$

The mentioned equations take the form after transformations ("System 2"):
(dimensionless continuity equation)

$$
\begin{equation*}
\frac{\partial \tilde{p}}{\partial \tilde{\xi}}-\tilde{F}=\tilde{C} \tag{2.9}
\end{equation*}
$$

(dimensionless momentum equation)

$$
\begin{equation*}
\tilde{C}-2 \tilde{C} \frac{\partial \tilde{u}}{\partial \tilde{\xi}}-3 \frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{p} \frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right\}+2 \frac{\partial}{\partial \tilde{\xi}}\left\{\tilde{u} \frac{\partial \tilde{p}}{\partial \tilde{\xi}}\right\}=0 \tag{2.10}
\end{equation*}
$$

(dimensionless energy equation)

$$
\begin{align*}
& 6 \tilde{u} \frac{\partial \tilde{p}}{\partial \tilde{\xi}}+5 \tilde{p} \frac{\partial \tilde{u}}{\partial \tilde{\xi}}+2 \tilde{u} \tilde{C} \\
& -16 \tilde{u} \frac{\partial}{\partial \tilde{\xi}}\left[\tilde{u} \frac{\partial \tilde{p}}{\partial \tilde{\xi}}\right]-26 \tilde{u} \frac{\partial}{\partial \tilde{\xi}}\left[\tilde{p} \frac{\partial \tilde{u}}{\partial \tilde{\xi}}\right]  \tag{2.11}\\
& -\left[10 \tilde{u} \frac{\partial \tilde{p}}{\partial \tilde{\xi}}+21 \tilde{p} \frac{\partial \tilde{u}}{\partial \tilde{\xi}}+8 \tilde{C} \tilde{u}\right] \frac{\partial \tilde{u}}{\partial \tilde{\xi}}=0
\end{align*}
$$

## Significant remarks:

1. The value $A^{\text {pert }}$ can be considered as an external fluctuation after appearance of mass perturbations like Higgs boson. But we should take into account self-oscillations of physical vacuum or flashes in PV.
2. Physical models based on Schrödinger - Madelung theory (like applied Ghirardi-Rimini-Weber model with massive flashes [5-11]) cannot be applied because Schrödinger - Madelung theory does not contain independent energy equation in principle.
3. The introduced constant $\tilde{C}$ is of principal significance defining a flash in PV. If $\tilde{C}=0$ we obtain only elementary solutions $\tilde{u}=$ const, $\tilde{p}=$ const, $\tilde{F}=0$.

## 3 Numerical Solution of System 2.

Now we are ready to display the results of the mathematical modeling realized with the help of Maple (the versions Maple 9 or more can be used). The system of equations (2.9) - (2.11) have great possibilities of mathematical modeling as a result of changing four Cauchy conditions and parameter $\tilde{C}$ describing the character features of initial perturbations which lead to the traveling wave formation.

Maple program contains Maple's notations - for example the expression $D(u)(0)=0$ means in the usual notations $(\partial \tilde{u} / \partial \tilde{\xi})(0)=0$, independent variable $t$ responds to $\tilde{\xi}$. The following Maple notations on figures are used: u- velocity $\tilde{u}$, p-pressure $\tilde{p}$, and f - the self-consistent force $\tilde{F}$. Explanations placed under all following figures. The results of the calculations are presented in figures $3.1-3.13$. The information required is contained in the figures and figure captions. We use for all calculations the Cauchy conditions

$$
\tilde{u}(0)=1, \quad \tilde{p}(0)=1, D(\tilde{u})(0)=0, D(\tilde{p})(0)=0
$$

which of course can be changed; parameter $\tilde{C}$ varies widely.


Figure 1. Evolution of $\tilde{u}(\tilde{\xi}), \tilde{p}(\tilde{\xi}), \tilde{F}(\tilde{\xi}) ; \tilde{C}=1$


Figure 2. Evolution of $\tilde{u}(\tilde{\xi}), \tilde{p}(\tilde{\xi}), \tilde{F}(\tilde{\xi}) ; \tilde{C}=10$


Figure 3. Evolution of $\tilde{u}(\tilde{\xi}), \tilde{p}(\tilde{\xi}) ; \tilde{C}=100$


Figure 4. Evolution of $\tilde{F}(\tilde{\xi}) ; \tilde{C}=100$


Figure 5. Evolution of $\tilde{u}(\tilde{\xi}), \tilde{p}(\tilde{\xi}) ; \tilde{C}=1000$


Figure 6. Evolution of $\tilde{F}(\tilde{\xi}) ; \tilde{C}=1000$


Figure 7. Evolution of $\tilde{u}(\tilde{\xi}) ; \tilde{C}=0$
$\mathrm{C}=-1, \lim 1=-0.958, \lim 2=0.302$, u-solid line, p -dashdot line, f-dotted line


Figure 8. Evolution of $\tilde{u}(\tilde{\xi}), \tilde{p}(\tilde{\xi}), \tilde{F}(\tilde{\xi}) ; \tilde{C}=-1$


Figure 9. Evolution of $\tilde{u}(\tilde{\xi}), \tilde{p}(\tilde{\xi}), \tilde{F}(\tilde{\xi}) ; \tilde{C}=-10$
$\mathrm{C}=-100$, lim $=-0.824$, u-solid line, p -dashdot line


Figure 10. Evolution of $\tilde{u}(\tilde{\xi}), \tilde{p}(\tilde{\xi}) ; \tilde{C}=-100$


Figure 11. Evolution of $\tilde{F}(\tilde{\xi}) ; \tilde{C}=-100$


Figure 12. Evolution of $\tilde{u}(\tilde{\xi}), \tilde{p}(\tilde{\xi}) ; \tilde{C}=-1000$


Figure 13. Evolution of $\tilde{F}(\tilde{\xi}) ; \tilde{C}=-1000$

## 4 Discussion and Conclusion.

1. As we see nonlocal physics can be applied to the description of transport processes in physical vacuum (PV). Local physics cannot be applied to PV.
2. Physical models based on Schrödinger - Madelung theory are not applicable to PV investigations.
3. Even the mass perturbations do not exist in PV, nevertheless the self-oscillations could lead to appearance of wave processes in PV. The originated waves propagate with the constant velocity in the case of strong flashes. In many cases the finite interval of possible $\tilde{\xi}$ values exists.
4. If mass perturbations $\tilde{A}^{\text {pert }}$ and flash perturbations $\tilde{C}$ are absent the wave processes do not exist (Fig. 7).
5. The self-consistent forces (acting on the volume unit) correspond to the both regimes - known as gravitation and anti-gravitation.
6. The special relativistic theory is a method to eliminate the influence of PV from consideration in closed thermodynamic systems.

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